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PARAREAL-RADIAL BASIS FUNCTION-FINITE DIFFERENCE (RBF-FD) FRAMEWORK FOR SOLVING TIME-DEPENDENT PARTIAL DIFFERENTIAL EQUATIONS

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This study combines the Parareal time solver algorithm with the Radial Basis Function-Finite Difference (RBF-FD) space discretisation to create a computationally efficient approach for solving large-scale partial differential equations (PDEs). The Parareal algorithm utilises a combination of coarse and fine standard ODE solvers to simultaneously approximate solutions at different time intervals. The RBF-FD is a mesh-free method used to discretise PDEs in space. The Polyharmonic Spline RBF is commonly used in RBF-FD computations; however, any other RBF could be used as well. The RBF methodology is similar to the method of lines approach, where the space discretisation would transform a time dependent PDE into a system of ordinary differential equations (ODEs). Therefore, to solve a time-dependent PDE, one could discretise the PDE in space using the RBF-FD method and use the Parareal algorithm to solve the system of ODEs. However, there are several issues when it comes to pairing the RBF-FD differentiation matrices with ODE solvers. Firstly, the RBF-FD differentiation matrices could have spurious eigenvalues that may fall outside the stability region of the ODE solver. To stabilise the algorithm, one needs to use unacceptably small-time steps, which increase computational costs. Secondly, the higher degree polynomials used in RBF-FD discretisation to achieve higher-order convergence increased the computational cost, especially in large-scale problems. The novelty of our work is that we address these issues by pairing a coarse and fine RBF-FD space discretisation with a respective coarse and fine time solver in the Parareal algorithm. We show that the coarse RBF-FD stencil preserves the higher-order convergence of the fine RBF-FD stencil through error analysis. Furthermore, we show the finite step convergence of the Parareal algorithm for various test cases, including the shallow water equations.

Keywords: Higher-order convergence, Parallel Time Solvers, Parareal, RBF-FD, Spurious Eigenvalues